Detection and countermeasure of interference in slow FH/MFSK systems over fading channels

Aye Aung *, Kah Chan Teh, Kwok Hung Li
School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore 639798, Singapore

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A B S T R A C T
In this paper, we present an algorithm to detect unknown interference in slow frequency-hopped M-ary frequency-shift-keying (SFH/MFSK) systems over fading channels. Both partial-band noise interference (PBNI) and multitone interference (MTI) are considered. The proposed algorithm performs the detection process after dehopping by making use of square-law detectors. We first analyze the statistical property of the outputs of the square-law detectors over one hop duration, and an appropriate threshold level is derived for detecting the interference based on a binary hypothesis testing. We also formulate the closed-form expressions for the probabilities of detection of both types of interference experienced in any particular frequency hop. The analytical results are validated by the simulation results and they reveal that the proposed algorithm is able to provide good detection performance for both types of interference and outperforms the conventional ratio–threshold test (RTT) method.

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1. Introduction

The use of frequency-hopped spread-spectrum (FH–SS) systems has attracted considerable interest in civilian and military wireless communications for highly secured data transmission. However, the performance of FH–SS systems can be unacceptably degraded by intentional/unintentional interference whose power is comparable with or much larger than the signal power [1,2]. Partial-band noise interference (PBNI) and multitone interference (MTI) are the two most frequently encountered interference waveforms in FH–SS systems [3–6]. Fast frequency-hopped systems may be used to protect against such deleterious interference by employing various diversity-combining techniques [7–9]. However, FH systems may have limitations for fast hopping [10]. In FH systems, the transmitters and receivers contain clocks which must be synchronized. The faster the hopping rate, the more accurate the clocks must be. In addition, high-end frequency synthesizer is required in order to generate the fast FH signal [1]. Therefore, some FH systems may still have restraints that do not allow for fast hopping.

In FH–SS systems, the presence of strong interference in the desired FH band will lead to degraded bit-error-rate (BER) performance of FH receivers, especially when the system is a slow FH–SS system where more than one symbol error can be introduced in one hop interval. It is also known that the receiver with perfect side information can achieve better BER performance [7,11,12]. In [10], it has been observed that the information on the presence or absence of the PBNI is essential for the proposed maximum-likelihood (ML)–based detector to reject the PBNI from the intentional interferers in slow FH M-ary frequency-shift-keying (MFSK) systems. In [13], the knowledge of interference has been shown to be useful to develop an interference-detection-assisted decoding scheme to further enhance the interference immunity in slow FH–SS systems. Therefore, the information of the presence or absence of detrimental interference is crucial to SFH–SS systems so that the interference
knowledge-aided FH receivers can be employed to maintain a reliable communication channel. In this paper, we will consider detection of unknown interference (PBNI or MTI) encountered in slow FH/MFSK systems. A common method for detecting a signal embedded in the background additive white Gaussian noise (AWGN) is to use the energy detector which compares the observed energy with a predefined threshold [14–17]. However, in our detection problem, the unknown interference will be present together with the desired FH signal plus noise at the receiver, which is different from the above-mentioned detection problems where the desired PBNI is present together with noise only. Therefore, these methods cannot be directly employed to detect the interference in FH–SS systems.

It is found that not much work has been done to detect interference in FH–SS systems. In [5,6], the authors used a ratio–threshold test (RTT) to enhance the system bit-error-rate (BER) performance of slow FH/MFSK systems. Interference was detected by the RTT before decoding the MFSK symbols. The RTT was performed on the largest and second largest outputs of \( M \) parallel square-law detectors. The corresponding ratio is compared with a threshold to decide the presence of the interference in each symbol. In [10], an ML-based algorithm which uses a two-element antenna array has been proposed for joint interference rejection and symbol detection in slow FH/MFSK systems. This ML-based algorithm requires the PBNI information on whether the interference is present in any particular hop duration. This information was obtained by deploying the directional of arrival (DOA) estimation-based smart antenna array nulling algorithm [18]. The antenna placed a null in the DOA of the desired signal and the interference is detected by monitoring the energy of the array output. However, this algorithm may not be able to detect the interference signal if the DOAs of both desired signal and interference are very close. The exponential averaging technique was proposed in [19] to detect the interference in FH–SS systems by utilizing the fast Fourier transform (FFT). Detection was performed in the frequency domain before dehopping the FH signal. Interference was detected by analyzing the averaged spectrum computed over a few hops duration. Therefore, it is not suitable for slow FH systems as it can cause a large processing delay. Besides, it will increase the amount of noise to the input of analog-to-digital converter (ADC) since sampling processing is done on a wideband SS signal before dehopping.

In this paper, we propose an algorithm to detect the unknown interference encountered in slow FH/MFSK systems by using a simple binary hypothesis testing [14]. The algorithm can be used to detect both types of interference, i.e., PBNI or MTI. The detection process is performed after dehopping; hence, the amount of noise to the input of ADC can be reduced as the sampling process is done only after the dehopped signal is filtered by the narrowband bandpass filter (BPF). By exploiting the statistical property of the outputs of the square-law detectors, a suitable threshold is derived, leading to interference detection in slow FH/MFSK systems. The closed-form expressions for probabilities of detection for both types of interference (PBNI or MTI) are also formulated. The rest of the paper is organized as follows. Section 2 describes the system model. The derivation of the proposed detection algorithm for the interference encountered in slow FH/MFSK systems over fading channels is presented in Section 3. Numerical results and discussions are provided in Section 4. Finally, this paper is concluded in Section 5.

## 2. System model

The conventional MFSK modulator selects one of the \( M \) baseband frequencies, \( f_m, m = 0, 1, \ldots, M - 1 \), based on the incoming data sequence with the symbol rate of \( R_s = 1/T_s \), where \( T_s \) is the symbol duration. The frequency separation between the adjacent signal tones is set to \( \Delta f = 1/T_s \), so the transmitted signals are orthogonal. In particular, we choose \( f_m = (m + 1)/T_s \), where \( m = 0, 1, \ldots, M - 1 \). Since it is a slow FH–SS system, more than one symbol are transmitted in one FH interval which is denoted as \( T_h \). Hence, \( L = T_h/T_s > 1 \) where \( L \) represents the number of transmitted symbols within one hop duration. The output of the MFSK modulator is then mixed by a frequency synthesizer to hop to one of the \( N_h \) nonoverlapping FH bands in which each FH band consists of \( M \) frequency slots for MFSK with their corresponding bandwidth of \( B = \Delta f \). The frequency synthesizer is then driven by a pseudo-noise (PN) code generator so that the hopping process is done in a pseudo-random manner. Finally, the signal is passed through a BPF with a spread spectrum bandwidth of \( W_{ss} \) and translated by a radio-frequency (RF) oscillator for transmission.

Fig. 1 illustrates a block diagram of the SFH/MFSK receiver with the proposed detection module. As shown in the figure, the received signal is first downconverted, band-pass \( (W_{ss}) \) filtered, and dehopped by a frequency synthesizer which is synchronously controlled by the same PN code used for the transmitter. Then the dehopped signal is passed through a BPF with narrowband bandwidth of \( W_d = M\Delta f \). Under the presence of AWGN and interference, the resultant signal of the \( k \)-th hop can be expressed as [7,8]

\[
y(t) = \sqrt{2}|a_k| \cos (2\pi f_m t + \theta_k) + n_w(t) + I(t)
\]

where \( \sqrt{2}|a_k| \) is the signal amplitude of the desired signal under fading channels with an averaged signal power of \( P_s = E[|a_k|^2] \) (where \( E[\cdot] \) denotes the expectation operator), \( \theta_k \) is the random phase for the \( k \)-th hop, which can be assumed to be uniformly distributed over \([0, 2\pi)\), \( f_m \) is the baseband signal frequency where \( m = 0, 1, \ldots, M - 1 \), \( n_w(t) \) is the noise term due to AWGN and it has a variance of \( \sigma_w^2 = N_0 W_d \), where \( N_0 \) is the one-sided power spectral density (PSD) of AWGN. The unknown interference \( I(t) \) is either PBNI or MTI. If \( I(t) \) is the PBNI, the strategy of interferer is to distribute its additive Gaussian noise power uniformly distributed over a fraction \( \rho \) of the total spread spectrum bandwidth \( W_{ss} \) where \( 0 < \rho \leq 1 \). The interferer is assumed to choose that the PBNI covers all the adjacent \( M \) slots, that is, the entire narrowband \( W_d \) if a hit or collision occurs for any particular hop [8]. Hence, \( I(t) \) has a variance of \( \sigma_I^2 = N_I W_d / \rho \), where \( N_I \) is the equivalent one-sided PSD of the full-band interference noise. We define the signal-to-noise power ratio to be \( \text{SNR} = P_s / \sigma_w^2 \) as well as
the signal-to-PBNI power ratio to be $\text{SPNR} = P_s/\sigma_i^2$. For the MTI, we shall consider a commonly used single-tone band MTI model [1,7,9], that is, only one tone is present in the desired FH band after dehopping once a collision occurs. Hence, it can be modeled as $I(t) = \sqrt{2}|a_i| \cos(2\pi f_i t + \phi_i)$ where $\sqrt{2}|a_i|$ is the amplitude of the tone interference under fading channels with an averaged power of $P_I = E(|a_i|^2)$, $\phi_i$ is the random phase which is uniformly distributed over $[0, 2\pi)$, and $f_i$ denotes the interference frequency which falls within the desired FH band. Consequently, we define the signal-to-tone-interference power ratio to be $\text{SIR} = P_s/P_I$ where $P_s$ is the power of the tone interference. The FH signal and tone interference are assumed to undergo the same fading environment.

3. Proposed detection algorithm

Fig. 2 shows the block diagram of the proposed detection algorithm in SFH/MFSK systems over fading channels. As previously described in Fig. 1, the signal $y(t)$ in Fig. 2 is first passed through a BPF with a bandwidth $W_0$. Hence, the interference outside the desired hop band will be filtered out. If a hit or collision occurs in any particular FH band, the interference (which hits the active FH band at any particular instant) will still be present in this FH band after dehopping. This proposed algorithm will determine whether such interference is present in any particular hop duration or not.

At the detection module, the signal is first detected by using $M$ pairs of correlators which are matched to frequencies $f_0, f_1, \ldots, f_{M-1}$, respectively. The output obtained from each pair of correlators is passed through a square-law device and a combiner in order to produce the following test statistic for the $k$-th hop:

$$T_k = \sum_{l=1}^{L} r_{kl}$$

where $r_{kl} = r_{kl}^{(0)} + r_{kl}^{(1)} + \cdots + r_{kl}^{(M-1)}$, $r_{kl}^{(m)} = z_{kl}^{(m)2}$ where $m = 0, 1, \ldots, M - 1$, $l = 0, 1, \ldots, L - 1$, the subscript $kl$ represents the $l$-th symbol of the $k$-th hop, and $L$ is the number of transmitted symbols in one frequency hopping interval. Each output $T_k$ is then compared with the threshold $\gamma$ to decide the presence of interference in any particular hopping interval. We first consider the PBNI for the purpose of deriving analytical expression of probability of detection ($P_D$). Two hypotheses are defined as follows:

$H_{k0}$ : Interference is not present in the $k$-th hop

$H_{k1}$ : Interference is present in the $k$-th hop.
In order to determine the threshold, the probability density function (PDF) of \( T_k \) under \( H_{k0} \) is required. We add the outputs of \( M \) square-law detectors over one hop period since better detection performance can be achieved if the overlap between the two PDFs under \( H_{k0} \) and \( H_{k1} \) becomes less [14]. Let \( r_k^{(S)} \) be the output of the square-law detector for the signal branch and \( r_k^{(NS)} \) be the output from the non-signal branch. Under fading channels, they can be expressed as follows [17,20]:

\[
r_k^{(S)} \sim \chi^2_2(\sigma^2, a^2_s) \tag{4}
\]

which is a noncentral chi-square random variable \( \chi^2_2 \) with two degrees of freedom, and its conditional pdf is given by

\[
p(r_k^{(S)}|a_i) = \frac{1}{2a^2} \exp\left(-\frac{r_k^{(S)} + a^2_s}{2\sigma^2}\right) I_0 \left(\frac{a\sqrt{r_k^{(S)}}}{\sigma^2}\right) U(r_k^{(S)}). \tag{5}
\]

For the non-signal branch,

\[
r_k^{(NS)} \sim \chi^2_2(\sigma^2) \tag{6}
\]

which is a central chi-square random variable with two degrees of freedom, and its pdf is provided by

\[
p(r_k^{(NS)}) = \frac{1}{2a^2} \exp\left(-\frac{r_k^{(NS)}}{2\sigma^2}\right) U(r_k^{(NS)}). \tag{7}
\]

where \( U(\cdot) \) is the unit step function, \( I_0(\cdot) \) is the zeroth-order modified Bessel function of the first kind, \( E(\cdot) \) is the average received signal power, and \( \sigma^2 \) is the variance of the quadrature noise components. The noise variance is \( \sigma^2 = \sigma_w^2 \) if no PBNI is present in this particular symbol, and \( \sigma^2 = \sigma_s^2 + \sigma_w^2 \) if the PBNI is present since the PBNI under fading channels can be treated like noise after dehopping [1,8,11]. The desired signal amplitude is modeled as a frequency-nonselective Rician fading random variable which is assumed to be constant over one symbol duration with its pdf given by [20]

\[
p(a_i) = \frac{a_i}{\sigma^2_s} \exp\left(-\frac{a_i^2 + 2\sigma_s^2}{2\sigma^2}\right) I_0 \left(\frac{a_i\alpha_s}{\sigma^2}\right) U(a_i). \tag{8}
\]

where the parameters \( \alpha^2_s \) and \( 2\sigma^2_s \) are the power levels of direct and diffused components of the desired signal, respectively. The total averaged power of the desired signal \( P_s = \alpha^2_s + 2\sigma^2_s \) is assumed to remain constant from hop to hop. The fading ratio \( K = \alpha^2_s / 2\sigma^2_s \) determines various levels of fading scenarios, e.g., essentially no fading (\( K = 30 \text{ dB} \)), Rician fading (\( K = 10 \text{ dB} \)) and Rayleigh fading (\( K = -\infty \text{ dB} \)). The unconditional PDF of \( r_k^{(S)} \) can be obtained by integrating (5) with respect to the PDF of \( a_i \) in order to produce

\[
p(r_k^{(S)}) = \frac{1}{2(\sigma^2 + \sigma^2_s)} \exp\left(-\frac{r_k^{(S)} + \sigma^2_s}{2(\sigma^2 + \sigma^2_s)}\right) I_0 \left(\frac{\alpha_s\sqrt{r_k^{(S)}}}{\sigma^2 + \sigma^2_s}\right) U(r_k^{(S)}). \tag{9}
\]

Hence,

\[
r_k^{(S)} \sim \chi^2_2(\sigma^2, a^2_s) \tag{10}
\]

In fact, the test statistic \( T_k \) in (2) under fading channels can be realized as a linear summation of \( L \) noncentral chi-square random variables and \( (M - 1) L \) central chi-square random variables as follows:

\[
T_k = \sum_{l=1}^{L} r_k^{(S)} + (M - 1) \sum_{l=1}^{L} r_k^{(NS)} = r_k^{(S)} + T_k^{(NS)}. \tag{11}
\]

Now let us define

\[
T_k^{(S)} = \sum_{l=1}^{L} r_k^{(S)} = z^H z \tag{12}
\]

where \((\cdot)^H\) represents the Hermitian transpose, and

\[
z = \begin{bmatrix} z_{k1} \\ z_{k2} \\ \vdots \\ z_{kL} \end{bmatrix}. \tag{13}
\]

Note that \( z_{kl} \) is a complex Gaussian random variable that is obtained from the summation of complex Gaussian variables whose real and imaginary parts are acquired from the cosine and sine branches of the correlators as shown in Fig. 2. Therefore, \( z_{kl} \) has a complex normal (CN) distribution as

\[
z_{kl} \sim \text{CN}(\mu_{z_{kl}}, \sigma^2_{z_{kl}}) \tag{14}
\]

where

\[
\mu_{z_{kl}} = E(\alpha_s e^{j\theta_{kl}}) \tag{15}
\]

and

\[
\sigma^2_{z_{kl}} = E\left((z_{kl} - \mu_{z_{kl}}) (z_{kl} - \mu_{z_{kl}})^*\right) = E\left(|z_{kl}|^2\right) - |\mu_{z_{kl}}|^2. \tag{16}
\]

Note that \( l = 1, 2, \ldots, L \) and \((\cdot)^*\) represents the complex conjugate. Now \( T_k^{(S)} \) can be seen as a quadratic form of complex-valued Gaussian vector \( z \) whose distribution is

\[
z \sim \text{CN}(\mu_z, \Sigma_z) \tag{17}
\]

where the mean and covariance of \( z \) are given by

\[
\mu_z = \begin{bmatrix} \mu_{z_{k1}} \\ \mu_{z_{k2}} \\ \vdots \\ \mu_{z_{kL}} \end{bmatrix} \tag{18}
\]

and

\[
\Sigma_z = \begin{bmatrix} \sigma^2_{z_{k1}} & \text{cov}(z_{k1} z_{k2}) & \cdots & \text{cov}(z_{k1} z_{kL}) \\ \text{cov}(z_{k1} z_{k2}) & \sigma^2_{z_{k2}} & \cdots & \text{cov}(z_{k2} z_{kL}) \\ \vdots & \vdots & \ddots & \vdots \\ \text{cov}(z_{k1} z_{kL}) & \text{cov}(z_{k2} z_{kL}) & \cdots & \sigma^2_{z_{kL}} \end{bmatrix}, \tag{19}
\]

respectively.

We will now determine the distribution of the random variable \( T_k^{(S)} \). Let us recall the central limit theorem.
which states that the asymptotic distribution of sums of independent and identically distributed random variables $X_1, X_2, \ldots, X_n$ is approximately normal-distributed with $N \sim \left( n \mu_X, \sigma_X^2 \right)$ for a large $n$ [21]. Hence, the idea is to transform the sum of correlated noncentral chi-square random variables to the sum of independent noncentral chi-square random variables, which in fact requires $z$ in (12) to be white. To do this, we first consider $z$ to be a result of transforming an $L$-dimensional complex Gaussian vector $g$ with zero mean and identity covariance matrix $I$ as follows [22]:

$$z = Q \left( g + Q^{-1} \mu_z \right)$$  \hspace{1cm} (20)

where $g \sim CN(0, I), Q$ represents the square root of $R$, i.e., $Q = R^{1/2}$ which can be obtained by the Cholesky decomposition of positive definite covariance matrix $R$, as

$$R = QQ^H.$$  \hspace{1cm} (21)

Next we find the eigenvalue decomposition of $R$, which yields

$$R = UAU^H$$  \hspace{1cm} (22)

since $R$ is the Hermitian matrix (hence, the normal matrix [23]) where $A = \text{diag} (\lambda_{k1}, \lambda_{k2}, \ldots, \lambda_{kL})$ denotes the diagonal matrix of the eigenvalues of $R$, and $U$ signifies the corresponding unitary matrix of the eigenvectors, which satisfies $UU^H = U^HU = I$. By substituting (20) into (12), $T_k$ can be obtained as follows:

$$T_k = (Q (g + Q^{-1} \mu_z))^H Q (g + Q^{-1} \mu_z)$$

$$= (g + Q^{-1} \mu_z)^H Q^H Q (g + Q^{-1} \mu_z).$$  \hspace{1cm} (23)

Since $Q^H Q = QQ^H$, by using (21) and (22), we can express $T_k$ in (23) as

$$T_k = (U^H g + U^H Q^{-1} \mu_z) A (U^H g + U^H Q^{-1} \mu_z)$$

$$= (h + \mu)^H A (h + \mu)$$  \hspace{1cm} (24)

where $h = U^H g$ and $\mu = U^H Q^{-1} \mu_z$.

Note that $h \sim CN(0, I)$ since the unitary transformation of $g$ does not change its distribution. Let $h_{kl}$ and $\mu_{kl}$ be the $l$-th entry of the $L$-dimensional vectors $h$ and $\mu$, respectively. Hence, $T_k$ in (24) can be rewritten as

$$T_k^{(S)} = \sum_{l=1}^{L} \lambda_{kl} |h_{kl} + \mu_{kl}|^2.$$  \hspace{1cm} (26)

Based on [24], $2 |h_{kl} + \mu_{kl}|^2$ is a noncentral chi-square random variable with two degrees of freedom and its noncentrality parameter is $2 |\mu_{kl}|^2$. Note that the multiplication with 2 is necessary since $h_{kl}$ is a zero-mean complex normal random variable with unit variance. Recall that if $z = x + jy$ is a complex Gaussian variable with $x \sim N(\sqrt{\sigma^2}, \sigma^2)$ and $y \sim N(\sqrt{\sigma^2}, \sigma^2)$, then $|z|^2 \sim \chi^2 (\sigma^2, P)$ [14,17]. Hence, we can further express the test statistic $T_k$ as

$$T_k^{(S)} = \frac{1}{2} \sum_{l=1}^{L} \xi_{kl}.$$  \hspace{1cm} (27)

where $\xi_{kl} = \chi^2 (\lambda_{kl}, 2\lambda_{kl} |\mu_{kl}|^2), I = 1, 2, \ldots, L$, are the independent random variables. Therefore, $T_k^{(S)}$ is nothing but a linear combination of independent noncentral chi-square random variables (with noncentral parameters $2\lambda_{kl} |\mu_{kl}|^2$) each having a distribution given by

$$p(\xi_{kl}) = \frac{1}{2 \lambda_{kl}} e^{-\frac{|I_{kl} + 2\lambda_{kl} |\mu_{kl}|^2}{\lambda_{kl}}} I_{0} \left( \sqrt{2\lambda_{kl} |\mu_{kl}|^2} \xi_{kl} \right).$$  \hspace{1cm} (28)

The mean and variance of the noncentral chi-square variable $\xi_{kl}$ are given by [17]

$$E[\xi_{kl}] = 2\lambda_{kl} + 2\lambda_{kl} |\mu_{kl}|^2$$  \hspace{1cm} (29)

and

$$\text{var}(\xi_{kl}) = 4\lambda_{kl}^2 + 8\lambda_{kl}^2 |\mu_{kl}|^2.$$  \hspace{1cm} (30)

As mentioned previously, by using the central limit theorem, we can use the Gaussian distribution to approximate the pdf of the test statistic $T_k^{(S)}$ which is a linear sum of independent noncentral chi-square random variables. Therefore,

$$T_k^{(S)} \sim N \left( \mu_{T_k^{(S)}}, \sigma_{T_k^{(S)}}^2 \right)$$  \hspace{1cm} (31)

where

$$\mu_{T_k^{(S)}} = \sum_{l=1}^{L} \lambda_{kl} (1 + |\mu_{kl}|^2)$$  \hspace{1cm} (32)

and

$$\sigma_{T_k^{(S)}}^2 = \sum_{l=1}^{L} \lambda_{kl}^2 (1 + 2 |\mu_{kl}|^2).$$  \hspace{1cm} (33)

In fact, we can express (32) and (33) in matrix forms as follows:

$$\mu_{T_k^{(S)}} = \text{Tr}(A) + (A \mu)^H \mu.$$  \hspace{1cm} (34)

$$\sigma_{T_k^{(S)}}^2 = \text{Tr}(A^2) + 2 (A^2 \mu)^H \mu.$$  \hspace{1cm} (35)

where $\text{Tr}(-)$ represents the trace of the matrix. We can further simplify (34) by substituting $\mu = U^H Q^{-1} \mu_z$, yielding

$$\mu_{T_k^{(S)}} = \text{Tr}(A) + \mu^H H \mu.$$  \hspace{1cm} (36)

In fact, $A$ is the diagonal matrix of the eigenvalues of $R$. Hence [23],

$$\text{Tr}(A) = \text{Tr}(R)$$  \hspace{1cm} (37)

which can be shown to be

$$\text{Tr}(R) = L \left( 2\sigma^2 + 2\sigma^2 \right)$$  \hspace{1cm} (38)

since $\xi_{kl}$ has a variance of $\sigma_{\xi_{kl}}^2 = 2\sigma^2 + 2\sigma^2$ with $\sigma^2 = \sigma^2 \under H_{kl}$ and $\sigma^2 = \sigma^2 + \sigma^2 \under H_{kl}$. Finally, (36) can be obtained as

$$\mu_{T_k^{(S)}} = L \left( 2\sigma^2 + 2\sigma^2 \right) + L \sigma^2$$

$$= 2L\sigma^2 + LP_s.$$  \hspace{1cm} (39)
where we had $\mu_2^H \mu_2 = L \alpha_s^2$, $\alpha_s^2$ and $2\sigma^2$ are the direct and diffused signal powers under Rician fading channels. Likewise, (35) can be further simplified as

$$\sigma^2_{t_k} = \text{Tr}(\Lambda^2) + 2\mu_2^H R_k \mu_2$$

(40)
after substituting $\mu = U^H Q^{-1} \mu_2$ and using $UU^H = I$ and $QQ^H = R_k$. Since $\Lambda$ is the diagonal matrix of the eigenvalues of $R_k$, we have [23]

$$\text{Tr}(\Lambda^2) = \text{Tr}(R_k^2)$$

(41)
which can be further simplified to

$$\text{Tr}(R_k^2) \approx L (2\sigma^2 + 2\alpha_s^2)^2$$

(42)
Subsequently, (40) can be obtained as

$$\sigma^2_{t_k} \approx L (2\sigma^2 + 2\alpha_s^2)^2 + 2\alpha_s^2 L (2\sigma^2 + 2\alpha_s^2)$$

(43)
where we use $\mu_2^H R_k \mu_2 \approx \alpha_s^2 L (2\sigma^2 + 2\alpha_s^2)$. For the non-signal branch, the PDF of $T_k^{(NS)}$ can be shown to be

$$T_k^{(NS)} \sim N \left( \mu_{t_k}^{(NS)}, \sigma^2_{t_k}^{(NS)} \right)$$

(44)
where the mean and variance of $T_k^{(NS)}$ are

$$\mu_{t_k}^{(NS)} = (M-1)L (2\sigma^2)$$

(45)
and

$$\sigma^2_{t_k}^{(NS)} = (M-1)L (4\sigma^4)$$

(46)
Finally, the PDF of $T_k$ can be obtained as

$$T_k \sim N \left( \mu_{t_k}, \sigma^2_{t_k} \right)$$

(47)
where

$$\mu_{t_k} = L (2\sigma^2 + 2\alpha_s^2) + L \alpha_s^2 + (M-1)L (2\sigma^2)$$

$$= 2ML\sigma^2 + LP_s$$

(48)
and

$$\sigma^2_{t_k} \approx L (2\sigma^2 + 2\alpha_s^2)^2 + 2\alpha_s^2 L (2\sigma^2 + 2\alpha_s^2)$$

$$+ (M-1)L (4\sigma^4)$$

(49)
Note again that $\sigma^2 = \sigma_w^2$ under $H_{10}$ and $\sigma^2 = \sigma_1^2 + \sigma_w^2$ under $H_{11}$. We will see later that the approximation in (49) is justified since the simulated and analytical values of $P_D$ are reasonably close to each other. We now have a detector which decides $H_{11}$ if

$$T_k > \gamma$$

(50)
and otherwise $H_{10}$, where $\gamma$ is a threshold predefined based on a given false alarm rate ($P_{FA}$) of the detector. Knowing the PDF expressions of the test statistic $T_k$ under both hypotheses, the detection threshold $\gamma$ can be configured based on $P_{FA}$ as [14]

$$P_{FA} = Q \left( \frac{\gamma - \mu_{t_{10}}^{(NS)}}{\sigma_{t_{10}}^{(NS)}} \right).$$

(51)
Hence,

$$\gamma = Q^{-1} (P_{FA}) \sigma_{t_{10}} + \mu_{t_{10}}$$

(52)
Note that $Q(\gamma) = \int_{-\gamma}^{\infty} \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{1}{2} t^2 \right) dt$, $\mu_{t_{10}}$ and $\sigma^2_{t_{10}}$ are respectively the mean and variance of the test statistic $T_k$ under $H_{10}$. The analytical expression of $P_D$ can be obtained as

$$P_D = Q \left( \frac{\gamma - \mu_{t_{11}}^{(NS)}}{\sigma_{t_{11}}^{(NS)}} \right)$$

(53)
where $\mu_{t_{11}}$ and $\sigma^2_{t_{11}}$ are the mean and variance of $T_k$ under $H_{11}$. The same threshold as defined in (52) can be used to detect the MTI as well. In order to derive the analytical expression of $P_D$, we consider that the tone interference falls within the desired FH band at one of the MFSK signal tones, i.e., $f_i = f_0$ if a hit occurs. It is also important to investigate the detection performance when $f_i = f_0$ since the FH signal and tone interference may add destructively or constructively. Under $H_{11}$, the resultant receiver power ($P_0$) can be expressed as [7]

$$P_0 = P_s + P_l + 2\sqrt{P_s P_l} \cos \Delta \varphi.$$ 

(54)
The random phase difference $\Delta \varphi$ between the signal tone and the interference tone is uniformly distributed over $(0, 2\pi]$. When the signal tone is shifted to $f_m$ where $m = 1, 2, \ldots, M - 1$, the received power becomes

$$P_l = P_s + P_l.$$ 

(55)
Since the probability of the occurrence of $f_m$ in FH/MFSK systems is assumed to be $1/M$, the conditional mean of $T_k$ under $H_{11}$ can be obtained as

$$\mu_{t_{11}} (\cos \Delta \varphi) = 2ML\sigma^2_w + (L/2)P_0 + (L/2)P_1$$

$$+ L\sqrt{P_s P_l} \cos \Delta \varphi$$

(56)
and the conditional variance of $T_k$ can be approximated as

$$\sigma^2_{t_{11}} (\cos \Delta \varphi) \approx \frac{L}{2} \left[ (2\sigma^2_w + 2\sigma^2_0)^2 + 2\alpha_s^2 (2\sigma^2_w + 2\sigma^2_0) \right]$$

$$+ \frac{L}{2} \left[ (2\sigma^2_w + 2\sigma^2_0)^2 + 2\alpha_s^2 (2\sigma^2_w + 2\sigma^2_0) \right]$$

$$+ (M-1)L (4\sigma^4).$$

(57)
Note that $\alpha_0^2$ and $2\sigma_0^2$ are the direct and diffused power levels based on $P_0$, and $\alpha_s^2$ and $2\sigma_s^2$ are the direct and diffused power levels based on $P_1$, respectively. The unconditional mean and variance can be acquired by

$$\mu_{t_{11}} = \int_0^\infty \mu_{t_{11}} (\cos \Delta \varphi) f (\Delta \varphi) \, d\Delta \varphi$$

(58)
and

$$\sigma^2_{t_{11}} = \int_0^\infty \sigma^2_{t_{11}} (\cos \Delta \varphi) f (\Delta \varphi) \, d\Delta \varphi,$$

(59)
where $f (\Delta \varphi) = 1/2\pi$. Finally, the analytical $P_D$ can be obtained as defined in (53).

The probabilities of detection for the PBN1 as well as tone interference have been derived in order to evaluate...
the detection performance of the proposed algorithm over fading channels. The decision on whether any particular hop is interfered or not can be made separately for each hop since the adjacent hops are independent of each other. In fact, we can make use of this interference detection algorithm to develop a suitable FH receiver to suppress the interference under the interference dominant environment over fading channels. As an example, this proposed detector can be used for the ML-based receiver proposed in [10] in order to provide the status of PBNI for any particular hop. Besides, this proposed algorithm can also be used to determine the set of FH channels that are interfered out of $N_h$ available FH channels. Accordingly, the system can modify the frequency hopping scheme to avoid transmission over these interfered FH channels [25,26]. As such the proposed detection algorithm can further improve the system error rate performance. In the next section, we will show the bit-error-rate (BER) simulation results by utilizing the proposed detector.

4. Numerical results and discussion

Numerical results are presented to show the detection performance of the proposed algorithm in slow FH/MFSK systems over fading channels. The analytical results are also provided in order to justify the detection performance demonstrated by the numerical results. We first consider $M = 2$, that is, slow FH/BFSK systems and subsequently, we also vary the value of $M$ to see how this can affect the detection performance. The parameters used for the SFH/BFSK system model are as follows: $R_s = 20$ kbps and $T_s = 50 \mu s$. Following that, the baseband frequencies for the BFSK modulated signal tones are $f_0 = 20$ kHz and $f_1 = 40$ kHz. The number of transmitted symbols within one hop duration is considered to be $L = 50$ so the frequency hopping interval is $T_h = 2.5$ ms. We also consider different values of $L$ later to investigate the detection performance of the algorithm. After dehopping, the received signal is filtered by an analog BPF having a bandwidth of $W_d = M \Delta f = 40$ kHz and passed through the detection module. In the square-law detectors, the discrete summation is used instead of the integration; hence, digital signal processors can be used to implement it. The sampling frequency we used is $F_s = 8R_s = 160$ kHz, and the number of digital samples used for discrete summation is $N = 8$. A higher symbol rate $R_s$ can be employed as well. Note that the detection performance will not be affected as long as the Nyquist sampling rate is used at the receiver (i.e., $F_s > 2W_d$). The averaged signal power $P_s$ and the noise power $\sigma_w^2$ are assumed to be known to the receiver in order to predetermine the threshold $\gamma$ as in (52). In practice, they can be obtained by the power spectrum estimation [27] during the initial measurement phase, where the interference is not activated as the receiver has the knowledge of $f_0$ and $f_1$. Simulations are run for 10,000 hops in which $L = 50$ symbols are transmitted in each FH interval, and the probability of detection is computed by using the threshold which is predefined based on the analysis.

In Fig. 3, we show the detection performance of the proposed algorithm with respect to increasing values of SPNR for various false alarm rates ($P_{FA}$) over Rayleigh fading channels. We can see that both analytical and simulated values of $P_D$ are remarkably close to each other, validating the detection performance analysis for our proposed scheme. As shown in the figure, the higher the value of $P_{FA}$, the better the detection performance is. In fact, one can choose a suitable value of $P_{FA}$ based on the system requirements. In this paper, we use $P_{FA} = 0.01$ for subsequent numerical results. It can also be seen that the probability of detection is reduced when SPNR increases, especially at about SPNR = 6 dB onwards. When SPNR is increased, the interference power becomes smaller as compared to the signal power. Hence, it will be more difficult to detect the PBNI. However, the PBNI has less

![Fig. 3. Analytical and simulated $P_D$ versus various SPNR levels under different values of $P_{FA}$ over Rayleigh fading channels ($M = 2$, SNR = 30 dB and $L = 50$).](image-url)
effect on the BER performance of the FH receivers when SPNR becomes larger, i.e., less interference power. Hence, it may not be critical to detect the PBNI under such situations. Note that even at SPNR = 6 dB, the proposed algorithm is able to detect the PBNI very well for any particular value of $P_{FA}$ since $P_D \approx 1$.

In Fig. 4, we investigate the detection performance of the proposed algorithm under various fading channel models. As shown in the figure, the detection performance under Rician fading ($K = 10$ dB) and AWGN (essentially no fading, $K = 30$ dB) is better than that under Rayleigh fading channels ($K = -\infty$ dB). In Fig. 5, we show how the detection performance for the PBNI is affected if the proposed algorithm is employed on different SFH/MFSK systems ($M = 2, 4, 8, 16, 32$). It is assumed that each system transmits the information at the same rate, i.e., $T_s = mT_b$ where $m = \log_2 M$, $T_b$ is the bit duration, and the PBNI covers the entire frequency-hopped band of $W_d = M\Delta f$. The interference power is assumed to remain constant over $W_d$. We can see from the figure that the detection performance is better when the modulation level $M$ is increased.

In Fig. 6, we illustrate the detection performance of the proposed scheme under different values of $L$ over Rayleigh fading channels. It can be seen that the detection performance is improved with the increase in $L$. The value of $L = 25$ also provides a reasonably good detection performance as well since $P_D \approx 1$ when SPNR = 4 dB. The smaller the value of $L$, the faster the hopping rate, and the higher the interference resistance. However, it will require a better frequency synthesizer as well as a more accurate clock to perform synchronization in both transmitters and
receivers. One may consider to use a suitable value of $L$ based on the system hardware constraints. We can also see the validity of our detection performance analysis for a small value of $L$ since the analytical and simulated values of $P_D$ are reasonably close to each other even for $L = 10$ over Rayleigh fading channels as shown in the figure.

In Fig. 7, we demonstrate the robustness of the proposed detection algorithm under various SNR conditions over Rayleigh fading channels. As shown in the figure, the detection performance degrades as the value of SNR decreases. It has been observed that even at the low SNR condition, where SNR is as low as 0 dB, the proposed scheme is able to detect the PBNI very well since $P_D \approx 1$ especially when the interference power is equal to the signal power (SPNR = 0 dB) as shown in the figure.

In Fig. 8, we show the detection performance of the algorithm for the tone interference over different fading channels when SNR = 30 dB. It is observed that the proposed algorithm is still able to detect the tone interference very well even though it is coincided with the BFSK signaling tone $f_0$ since $P_D \approx 1$ when $\text{SIR} = 0$ dB over different fading channels. Besides, the close match between the simulated and analytical values of $P_D$ indicates the validity of our analytical expression derived for the tone interference as well. The robustness of the proposed algorithm under different SNR levels over Rayleigh fading channels is further investigated in Fig. 9. As depicted in the figure, the detection performance deteriorates when the SNR level is reduced. It is found that the proposed algorithm is able to function well under low SNR conditions since $P_D \approx 89\%$ even at SNR = 5 dB, especially when SIR = 0 dB. However, the detection performance is reduced to $P_D = 36.3\%$ when SNR = 0 dB.
For the purpose of comparison, we consider ratio–threshold test (RTT) [6] which is able to detect the interference in each symbol. The optimal value of the threshold $\theta$ for the RTT is varied from 0.8 to 0.86. We use $\theta = 0.8$ since a higher value of $\theta$ will reduce the detection performance. For detection comparison in the hop level, the RTT decides the presence of interference in any particular hop if the interference is detected in any $p$ symbols (or more) out of $L$ symbols per hop. We consider $p = 3$ in our simulation. Fig. 10 shows the performance comparison for the PBNI encountered in slow FH/32-aryFSK systems over Rayleigh fading channels. As shown in the figure, the proposed algorithm is able to provide better detection performance than the RTT since $P_D \approx 99\%$ even when $\text{SNR}_{\text{PBNI}} = 20\text{ dB}$ whereas RTT is able to provide only $P_D \approx 65\%$. Intuitively, it can be explained that adding the outputs of square-law detectors can provide better interference detection than taking a ratio between the largest and second largest outputs. Fig. 11 depicts performance comparison for the MTI. As illustrated in the figure, the proposed algorithm is able to provide $P_D = 1$ when $\text{SIR} \leq 0\text{ dB}$ whereas the RTT does not perform well. For the MTI, the interference and FH signal tones can add constructively or destructively depending on their phases. Hence, the RTT cannot detect the MTI if these two tones are adding constructively. As a result, the detection performance is found to be worse for the RTT method. However, the proposed algorithm is still able to detect tone interference well under such situations as shown in the figure. We further observe that the RTT is able to outperform the proposed algorithm when $\text{SIR} > 2\text{ dB}$.

In Figs. 12 and 13, we illustrate how the proposed detector can benefit our slow FH/BFSK system in terms of system bit-error-rate (BER) performance by avoiding transmission over the interfered FH channels. We consider the number of available FH bands to be $N_h = 32$. Hence, the total spread spectrum bandwidth is $W_{ss} = 32W_d\text{ Hz}$. It is assumed that half of $W_{ss}\text{ Hz}$ is interfered by the

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**Fig. 8.** Analytical and simulated $P_D$ versus various SIR levels for the tone interference over different fading channels ($M = 2$, SNR = 30 dB and $L = 50$).

**Fig. 9.** $P_D$ vs. SIR for the tone interference under different SNR levels over Rayleigh fading ($M = 2$, $L = 50$).
Fig. 10. Performance comparison for the PBNI over Rayleigh fading channels ($M = 32$, $L = 50$ and SNR = 30 dB).

Fig. 11. Performance comparison for the MTI over Rayleigh fading channels ($M = 32$, $L = 50$ and SNR = 30 dB).

stationary PBNI or MTI. The BER simulation is conducted by transmitting 10,000 hops each consisting of 50 symbols ($L = 50$) over Rayleigh fading channels under the presence of interference, and the results are shown in Figs. 12 and 13 for the PBNI and MTI, respectively, for the two FH receivers, both with and without the proposed detector. We can see from the figures that a significant improvement in BER performance can be achieved for the receiver employing the proposed detector especially when the interference power level is equal to the signal power level, i.e., SPNR = 0 dB for the PBNI, and SIR = 0 dB for the MTI. However, when SPNR and SIR increase, the detection performance for the interference degrades. This in turn reduces the BER performance of the FH receiver, whose BER curves become closer to the BER curves without the interference detector as shown in the figures.

5. Conclusion

A detection algorithm for unknown interference which is commonly encountered in slow FH/MFSK systems over fading channels has been proposed in this paper. Both PBNI and MTI have been considered. By exploiting the statistical property of the outputs of the square-law detectors over
one hop duration, a suitable threshold has been derived in order to develop the proposed scheme. The analytical expressions for probabilities of detection ($P_D$) of the PBNI and MTI have been derived for any particular hop. The analytical results have been validated by the simulation results and it has been shown that the proposed algorithm is able to detect both types of interference very well. As compared with the RTT, the proposed algorithm is able to provide better detection performance. We have also shown that with the aid of the proposed interference detector, the system BER performance can be improved by avoiding transmission over the interfered FH channels. In terms of computational complexity, the algorithm possesses a low computational cost since it only requires the addition of the outputs of the square-law detectors to make a decision for detection. For the future work, we will further study suitable interference mitigation algorithms based on the proposed interference detector in slow FH–SS systems over fading channels.

References


